MARKSCHEME

May 2006

MATHEMATICS

Standard Level

Paper 2

Instructions to Examiners

Abbreviations

M   Marks awarded for attempting to use a correct **Method**; working must be seen.

(M)   Marks awarded for **Method**; may be implied by **correct** subsequent working.

A   Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding *M* marks.

(A)   Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R   Marks awarded for clear **Reasoning**.

N   Marks awarded for **correct** answers if **no** working shown (or working which gains no other marks).

AG   Answer given in the question and so no marks are awarded.

Using the markscheme

1   General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations *M1*, *A1*, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2   Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award *M0* followed by *A1*, as *A* mark(s) depend on the preceding *M* mark(s), if any.
- Where *M* and *A* marks are noted on the same line, *e.g.* *M1A1*, this usually means *M1* for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and *A1* for using the **correct** values.
- Where the markscheme specifies *(M2)*, *N3*, *etc*., do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3   *N* marks

*Award N marks for **correct** answers where there is **no** working, (or working which gains no other marks).*

- Do **not** award a mixture of *N* and other marks.
- There may be fewer *N* marks available than the total of *M*, *A* and *R* marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- For consistency within the markscheme, *N* marks are noted for every part, even when these match the mark breakdown. In these cases, the marks may be recorded in either form *e.g. A2* or *N2*.

4   Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5   Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6   Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then write –1(**MR**) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalised only once for a particular mis-read.*

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7   Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8   Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9   Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad\qquad \boldsymbol{A1}$$

Award *A1* for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10   Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

*Candidates should be penalized **once only IN THE PAPER** for an accuracy error (AP).   Award the marks as usual then write (AP) against the answer. On the **front** cover write –1(AP). Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the *AP*.
- If the level of accuracy is not specified in the question, apply the *AP* for correct answers not given to three significant figures.
- Intermediate values are sometimes written as 3.24(741). This indicates that using 3.24 (or 3.25) is acceptable, but the more accurate value is 3.24741. The digits in brackets are not required for the marks. If candidates work with fewer than three significant figures, this could lead to an *AP*.

If there is no working shown, and answers are given to the correct two significant figures, apply the *AP*. However, do **not** accept answers to one significant figure without working.

11   Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12   Examples

Exemplar material is available in the examiner training on Webct. Please refer to this material before you start marking, and when you have any queries. Please also feel free to contact your Team Leader if you need further advice.

QUESTION 1

(a) (i) $f'(x) = -\dfrac{3}{2}x + 1$ *A1A1*   *N2*

(ii) For using the derivative to find the gradient of the tangent   *(M1)*

        $f'(2) = -2$ *(A1)*

        Using negative reciprocal to find the gradient of the normal $\left(\dfrac{1}{2}\right)$ *M1*

        $y - 3 = \dfrac{1}{2}(x - 2)$ $\left(\text{or } y = \dfrac{1}{2}x + 2\right)$ *A1*   *N3*

(iii) Equating $-\dfrac{3}{4}x^2 + x + 4 = \dfrac{1}{2}x + 2$ (or sketch of graph)   *M1*

        $3x^2 - 2x - 8 = 0$ *(A1)*

        $(3x + 4)(x - 2) = 0$

        $x = -\dfrac{4}{3}$ $(=-1.33)$ $\left(\text{accept } \left(-\dfrac{4}{3}, \dfrac{4}{3}\right) \text{ or } x = -\dfrac{4}{3}, x = 2\right)$ *A1*   *N2*

                                                                        [9 marks]

(b) (i) Any **completely** correct expression (accept absence of dx)   *A2*

        e.g. $\displaystyle\int_{-1}^{2}\left(-\dfrac{3}{4}x^2 + x + 4\right)dx$, $\left[-\dfrac{1}{4}x^3 + \dfrac{1}{2}x^2 + 4x\right]_{-1}^{2}$ *N2*

(ii) Area $= \dfrac{45}{4}$ $(=11.25)$ (accept 11.3)   *A1*   *N1*

(iii) Attempting to **use** the formula for the volume   *(M1)*

        e.g. $\displaystyle\int_{-1}^{2}\pi\left(-\dfrac{3}{4}x^2 + x + 4\right)^2 dx$, $\pi\displaystyle\int_{-1}^{2}\left(-\dfrac{3}{4}x^2 + x + 4\right)^2 dx$ *A2*   *N3*

                                                                        [6 marks]

(c) $\displaystyle\int_{1}^{k} f(x)\,dx = \left[-\dfrac{1}{4}x^3 + \dfrac{1}{2}x^2 + 4x\right]_{1}^{k}$ *A1A1A1*

> **Note:** Award *A1* for $-\dfrac{1}{4}x^3$, *A1* for $\dfrac{1}{2}x^2$, *A1* for $4x$.

    Substituting $\left(-\dfrac{1}{4}k^3 + \dfrac{1}{2}k^2 + 4k\right) - \left(-\dfrac{1}{4} + \dfrac{1}{2} + 4\right)$ *(M1)(A1)*

    $= -\dfrac{1}{4}k^3 + \dfrac{1}{2}k^2 + 4k - 4.25$ *A1*   *N3*

                                                                        [6 marks]

                                                                        Total [21 marks]

QUESTION 2

(a) For **correct** substitution into cosine rule *A1*

$$BD = \sqrt{4^2 + 8^2 - 2 \times 4 \times 8 \cos\theta}$$

For factorising 16, $BD = \sqrt{16(5 - 4\cos\theta)}$ *A1*

$$= 4\sqrt{5 - 4\cos\theta}$$ *AG*   *N0*

[2 marks]

(b) (i) $BD = 5.5653 \ldots$ *(A1)*

$$\frac{\sin C\hat{B}D}{12} = \frac{\sin 25}{5.5653}$$ *M1A1*

$\sin C\hat{B}D = 0.911$ (accept 0.910, subject to *AP*) *A1*   *N2*

(ii) $C\hat{B}D = 65.7°$ *A1*   *N1*

Or $C\hat{B}D = 180 -$ their acute angle *(M1)*

$$= 114°$$ *A1*   *N2*

(iii) $B\hat{D}C = 89.3°$ *(A1)*

$$\frac{BC}{\sin 89.3} = \frac{5.5653}{\sin 25} \text{ or } \frac{BC}{\sin 89.3} = \frac{12}{\sin 65.7} \text{ (or cosine rule)}$$ *M1A1*

$BC = 13.2$ (accept 13.17…) *A1*

Perimeter $= 4 + 8 + 12 + 13.2$

$$= 37.2$$ *A1*   *N2*

[12 marks]

(c) Area $= \dfrac{1}{2} \times 4 \times 8 \times \sin 40°$ *A1*

$$= 10.3$$ *A1*   *N1*

[2 marks]

Total [16 marks]

QUESTION 3

Part A

(a) Adding probabilities   *(M1)*
Evidence of knowing that sum $=1$ for probability distribution   *R1*
e.g. Sum greater than 1, sum $=1.3$, sum does not equal 1   *N2*
[2 marks]

(b) Equating sum to 1 ($3k + 0.7 = 1$)   *M1*
$$k = 0.1$$ *A1*   *N1*
[2 marks]

(c) (i) $\text{P}(X = 0) = \dfrac{0+1}{20}$ *(M1)*

$$= \dfrac{1}{20}$$ *A1*   *N2*

(ii)   Evidence of using $\text{P}(X > 0) = 1 - \text{P}(X = 0)$ $\left(\text{or } \dfrac{4}{20} + \dfrac{5}{20} + \dfrac{10}{20} \right)$ *(M1)*

$$= \dfrac{19}{20}$$ *A1*   *N2*

[4 marks]

Sub-total [8 marks]

Part B

(a)
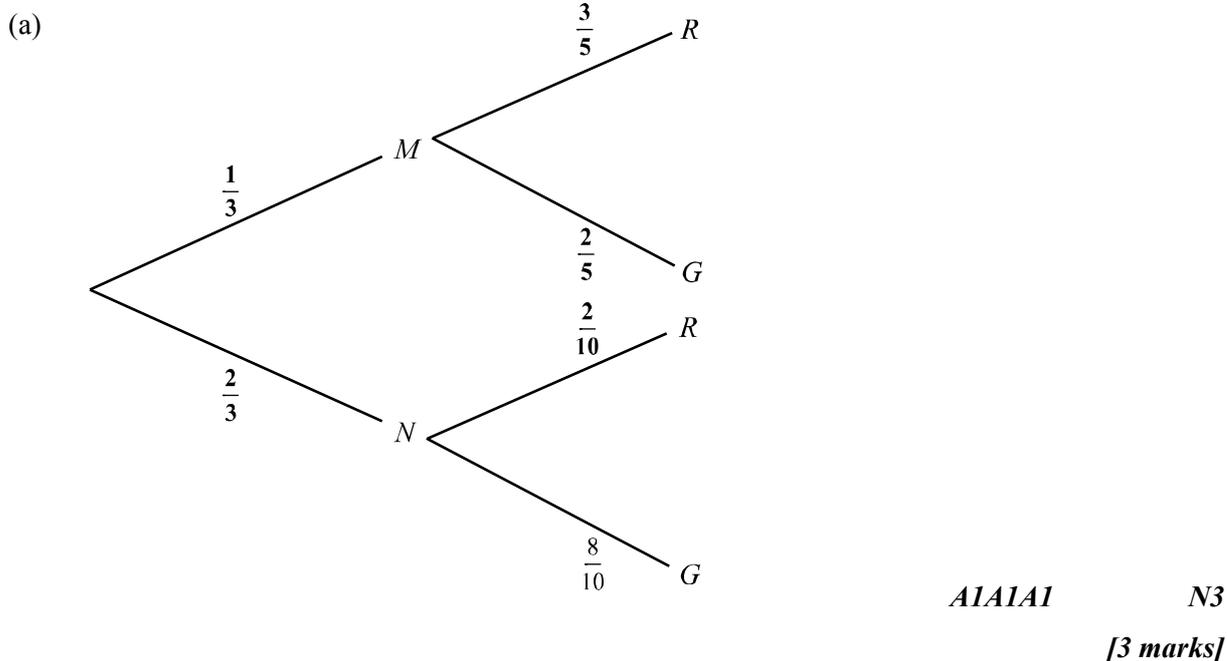
A1A1A1   *N3*

[3 marks]

Question 3 Part B continued

(b) (i) $P(M \text{ and } G) = \frac{1}{3} \times \frac{2}{5} (= \frac{2}{15} = 0.133)$ *A1*   *N1*

(ii) $P(G) = \frac{1}{3} \times \frac{2}{5} + \frac{2}{3} \times \frac{8}{10}$ *(A1)(A1)*

$= \frac{10}{15} \quad \left(= \frac{2}{3} = 0.667 \right)$ *A1*   *N3*

(iii) $P(M \mid G) = \frac{P(M \cap G)}{P(G)} = \frac{\frac{2}{15}}{\frac{2}{3}}$ *(A1)(A1)*

$= \frac{1}{5} \text{ or } 0.2$ *A1*   *N3*

[7 marks]

(c) $P(R) = 1 - \frac{2}{3} = \frac{1}{3}$ *(A1)*

Evidence of using a correct formula   *M1*

$E(\text{win}) = 2 \times \frac{1}{3} + 5 \times \frac{2}{3} \quad \left(\text{or } 2 \times \frac{1}{3} \times \frac{3}{5} + 2 \times \frac{2}{3} \times \frac{2}{10} + 5 \times \frac{1}{3} \times \frac{2}{5} + 5 \times \frac{2}{3} \times \frac{8}{10} \right)$ *A1*

$= \$4 \quad \left(\text{accept } \frac{12}{3}, \frac{60}{15} \right)$ *A1*   *N2*

[4 marks]

Sub-total [14 marks]

Total [22 marks]

QUESTION 4

(a) (i) $r = -2$ *A1*   *N1*

(ii) $u_{15} = -3(-2)^{14}$ *(A1)*

$= -49152$ (accept $-49\,200$) *A1*   *N2*

[3 marks]

(b) (i)   2, 6, 18 *A1*   *N1*

(ii) $r = 3$ *A1*   *N1*

[2 marks]

(c) Setting up equation (or a sketch) *M1*

$\dfrac{x+1}{x-3} = \dfrac{2x+8}{x+1}$ (or correct sketch with relevant information) *A1*

$x^2 + 2x + 1 = 2x^2 + 2x - 24$ *(A1)*

$x^2 = 25$

$x = 5$ or $x = -5$

$x = -5$ *A1*   *N2*

> **Note:** If "trial and error" is used, work must be documented with several trials shown.
> Award full marks for a correct answer with this approach.
> If the work is **not** documented, award *N2* for a correct answer.

[4 marks]

(d) (i) $r = \dfrac{1}{2}$ *A1*   *N1*

(ii) For attempting to use infinite sum formula for a GP *(M1)*

$S = \dfrac{-8}{1 - \dfrac{1}{2}}$

$S = -16$ *A1*   *N2*

> **Note:** Award *M0A0* if candidates use a value of r where $r > 1$, or $r < -1$.

[3 marks]

Total [12 marks]

QUESTION 5

(a) (i)   Evidence of subtracting all three components in the correct order   *M1*

$$e.g.\ \vec{AB} = \vec{OB} - \vec{OA} = (4\boldsymbol{i} - 5\boldsymbol{j} + 21\boldsymbol{k}) - (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k})$$
$$= 2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k}$$ *AG*   *N0*

(ii) $\left|\vec{AB}\right| = \sqrt{2^2 + (-8)^2 + 20^2}\ \ \left(= \sqrt{468} = 6\sqrt{13} = 2\sqrt{117} = 21.6\right)$ *(A1)*

$$\boldsymbol{u} = \frac{1}{\sqrt{468}}(2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k})$$ *A1*   *N2*

$$\left(= \frac{2}{\sqrt{468}}\boldsymbol{i} - \frac{8}{\sqrt{468}}\boldsymbol{j} + \frac{20}{\sqrt{468}}\boldsymbol{k},\ 0.0925\boldsymbol{i} - 0370\boldsymbol{j} + 0.925\boldsymbol{k},\ etc.\right)$$

(iii) If the scalar product is zero, the vectors are perpendicular.   *R1*

Note: Award *R1* for stating the relationship between the scalar product and perpendicularity, seen anywhere in the solution.

Finding an appropriate scalar product $\left(u \bullet \vec{OA},\ \text{or}\ \vec{AB} \bullet \vec{OA}\right)$ *M1*

$$e.g.\ u \bullet \vec{OA} = \left(\frac{2}{\sqrt{468}}\right) \times 2 + \left(\frac{-8}{\sqrt{468}}\right) \times 3 + \left(\frac{20}{\sqrt{468}}\right) \times 1\ \ \left(= \frac{4 - 24 + 20}{\sqrt{468}}\right)$$

$$\vec{AB} \bullet \vec{OA} = 2 \times 2 + (-8) \times 3 + 20 \times 1$$

$$\boldsymbol{u} \bullet \vec{OA} = 0\ \ \text{or}\ \ \vec{AB} \bullet \vec{OA} = 0$$ *A1*   *N0*

[6 marks]

(b) (i)   **EITHER**

$$S\left(\frac{2+4}{2}, \frac{3-5}{2}, \frac{1+21}{2}\right)$$ *(M1)(A1)*

Therefore, $\vec{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ (accept (3, −1, 11))   *A1*   *N3*

OR

$$\vec{OS} = \vec{OA} + \frac{1}{2}\vec{AB}$$ *(M1)*

$$= (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}) + \frac{1}{2}(2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k})$$ *(A1)*

$$\vec{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$$ *A1*   *N3*

(ii) $L_1 : \boldsymbol{r} = (3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}) + t(2\boldsymbol{i} + 3\boldsymbol{j} + 1\boldsymbol{k})$ *A1*   *N1*

[4 marks]

Question 5 continued

(c)   Using direction vectors (*e.g.* $2i + 3j + 1k$ and $-2i + 5j - 3k$)   **(M1)**

      Valid explanation of why L_1 is not parallel to L_2 **R1**   **N2**

        e.g.   Direction vectors are not scalar multiples of each other.
               Angle between the direction vectors is not zero or 180.
               Finding the angle

               $d_1 \cdot d_2 \neq |d_1||d_2|$.

> **Note:**   Award **R0** for "direction vectors are not equal".

                                                                      [2 marks]

(d)   Setting up any **two** of the three equations   **(M1)**
      For each correct equation   **A1A1**
      e.g. $3 + 2t = 5 - 2s$, $-1 + 3t = 10 + 5s$, $11 + t = 10 - 3s$

      Attempt to solve these equations   **(M1)**
      Finding **one** correct parameter $(s = -1,\ t = 2)$ **(A1)**

      P has position vector $7i + 5j + 13k$ **A2**   **N4**

> **Note:**   Award **(M1)A2** if the same parameter is used for both lines in the initial correct equations.
>   Award no further marks.

                                                                      [7 marks]

                                                                Total [19 marks]